Games, graphs, and machines

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Function or not?

Do the following rules define functions?

1. $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(s) = s^2$. \checkmark not injective 2. $f: \mathbb{Z} \to \mathbb{Z}$ defined by f(s) = s/2. 3. $f: \mathbb{Z} \to \mathbb{R}$ defined by f(s) = s/2. 4. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(s) = \sqrt{s}$. $X \quad f: \mathbb{R} \to \mathbb{C}$ 5. $f : \mathbb{R} \to \mathbb{R}$ defined by Still no \times $\underbrace{F(x):}_{O} : f(s) = \begin{cases} 1 \text{ if } s > 0 & & \\ 1 \text{ if } s < 0 & \\ 1 \text{ if } s < 0 & \\ \end{cases}.$ because is ambiguous. (2) Change domain to R-303

$$f: A \rightarrow B \quad inj \implies |A| \le |B|$$

$$hot \quad consversely.$$

$$A = \{1, 2, 3\} \quad B = \{1, 2, 3, 4\}$$

$$f: 1 \implies 1$$

$$2 \implies 2$$

$$f: A \rightarrow B$$

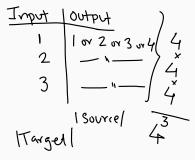
$$3 \implies 2$$

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

1. How many functions are there from A to B?

2. How many of these are injective functions?

3. How many of these are surjective functions?



Suppose A has size n and B has size m. How many functions are there from A to B? $\gamma \gamma$ Suppose $n \leq m$. How many functions are injective?

Suppose $f: S \to T$ is a bijection.

The inverse of f is the function $g: T \to S$ defined by the property that if t = f(s) then s = g(t).

- 1. Suppose $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse $g: : \backslash \sqcup \Im 2 \sqcup \backslash 3 \sqcup 2$
- 3. Find a function $f : \mathbb{R} \to \mathbb{R}$ which is its own inverse.

Consider $R \subset \mathbb{R} \times \mathbb{R}$ defined by

$$R = \{(x, y) \mid x^3 - xy + x - 1 = 0\}.$$

Is *R* the input/output relation of a function $f : \mathbb{R} \to \mathbb{R}$?